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AN APPROXIMATE METHOD OF DERIVING LOITER TIME FROM RANGE

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Nomenclature

C	= Specific Fuel Consumption (fuel flow rate per unit thrust produced)
C_P	= Power-Specific Fuel Consumption (fuel flow rate per unit power)
E	= Endurance (loiter time)
L/D	= Lift-to-Drag Ratio
R	= Range
V	= Velocity
W_f	= Aircraft Final Weight (after cruise or loiter mission segment)
W_i	= Aircraft Initial Weight (before cruise or loiter mission segment)
η_p	= Propeller efficiency parameter

Introduction

With the advent of the information warfare age, there is a greater need for long-loitering aircraft carrying radars, sensors, communication gear, and the like. Since cost is an always-present constraint, the aircraft platforms for such roles are often converted from other missions such as commercial transportation. In preliminary studies of such conversions, the available loiter time of various platforms is a key parameter for mission feasibility assessment and platform selection. While range and cruise speed are readily available for most such aircraft, the mid-mission loiter time usually is not.

This Note describes the derivation and use of a simple relationship between range and endurance based on the Breguet range and loiter equations. Given a known aircraft range and cruise speed, equivalent loiter time can be estimated with reasonable accuracy. In a typical application this relationship will allow rapid estimation of on-station endurance of commercial aircraft converted to military patrol usage such as Antisubmarine Warfare or Airborne Early Warning.

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Derivation of Method - Jet

The Breguet equations are commonly used analytical estimation methods for range and loiter, as derived in Raymer[1] and shown below as equations (1) and (2). These equations assume that the aircraft is allowed to slowly climb as fuel is burned off and weight is reduced (called a "cruise-climb" in the case of range). This is required to maintain the assumption of constant lift coefficient used in their derivation, which also tends to provide the maximum possible range or loiter time.

Range:

$$R = \frac{V}{C}\frac{L}{D} \ln\left(\frac{Wi}{Wf}\right) \tag{1}$$

Endurance:

$$E = \frac{1}{C}\frac{L}{D} \ln\left(\frac{Wi}{Wf}\right) \tag{2}$$

The endurance equation (2) appears to be nothing more than the range equation (1) divided by the velocity. This is trivially obvious from dimensional analysis, and for cruising flight this division correctly gives you the total flight time over that distance. However, we optimally loiter an aircraft at a lower speed than the speed for best range so this simple relationship must be adjusted both for aerodynamic (*L/D*) and propulsion (*C*) changes due to the differing speed.

In equation (3) below, we divide equation (1) by velocity. However, equation (1) uses *L/D* and *C* at cruise conditions, so we employ ratios to allow usage of loiter conditions.

$$\frac{R_{cruise}}{V_{cruise}} = \left(\frac{1}{C_{loiter}}\right)\left(\frac{C_{loiter}}{C_{cruise}}\right)\left(\frac{L}{D}\right)_{loiter}\left(\frac{\left(\frac{L}{D}\right)_{cruise}}{\left(\frac{L}{D}\right)_{loiter}}\right) \ln\left(\frac{Wi}{Wf}\right) \tag{3}$$

$$\frac{R_{cruise}}{V_{cruise}} = \left(\frac{C_{loiter}}{C_{cruise}}\right)\left(\frac{\left(\frac{L}{D}\right)_{cruise}}{\left(\frac{L}{D}\right)_{loiter}}\right)\left\{\left(\frac{1}{C_{loiter}}\right)\left(\frac{L}{D}\right)_{loiter} \ln\left(\frac{Wi}{Wf}\right)\right\} \tag{4}$$

or,

$$\frac{R_{cruise}}{V_{cruise}} = \left(\frac{C_{loiter}}{C_{cruise}}\right)\left(\frac{\left(\frac{L}{D}\right)_{cruise}}{\left(\frac{L}{D}\right)_{loiter}}\right)\left\{E_{loiter}\right\} \tag{5}$$

In equation (4) we have collected terms such that the large bracket at the right is now the desired endurance (E), with aerodynamic and propulsion terms under loiter conditions (eq. 5). Solving for this loiter time gives:

$$E_{loiter} = \left(\frac{R_{cruise}}{V_{cruise}}\right) \Bigg/ \left(\left(\frac{C_{loiter}}{C_{cruise}}\right)\left(\frac{\left(\frac{L}{D}\right)_{cruise}}{\left(\frac{L}{D}\right)_{loiter}}\right)\right) \tag{6}$$

Now we need to estimate those propulsion and aerodynamic ratios from cruise to loiter conditions.

The change in speed from cruise to loiter changes the aircraft lift-to-drag ratio since it changes the lift coefficient. It can be theoretically shown (see Raymer[1]) that a jet loiters optimally at its speed for best L/D, whereas it cruises optimally at a higher speed. At this higher speed the aircraft is flying at a condition where the ratio $C_L^{1/2}/C_D$ is maximized. This higher speed results in an L/D that is reduced to 0.866 of the optimal L/D (the aircraft goes further nonetheless because of the higher speed).

$$\left(\frac{L}{D}\right)_{cruise} = .866\left(\frac{L}{D}\right)_{loiter} \tag{7}$$

Substituting this into equation (6) gives:

$$E_{loiter} = \frac{1.16\frac{R_{cruise}}{V_{cruise}}}{\left(\frac{C_{loiter}}{C_{cruise}}\right)} \tag{8}$$

This can be used for jet aircraft where the engine fuel consumption data under cruise and loiter conditions is known (for propeller-powered aircraft, see below). For most jet engines, the specific fuel consumption (C) is slightly improved at the lower speeds of loiter. However, throttle setting is reduced for loiter which for many jet engines results in an increase in fuel consumption so perhaps the change in specific fuel consumption can be ignored, resulting in equation (9).

$$E_{loiter} = 1.16 \frac{R_{cruise}}{V_{cruise}} \qquad \text{(approximation for most jets)} \qquad (9)$$

Obviously, consistent units must be employed, such as range in nautical miles and speed in knots, resulting in loiter time in hours.

A check of this method for jets was made using the Boeing E-6A, for which range and endurance data is publicly available[2]. The range is 6,350 nmi with a 455 kt cruising speed. Its total endurance is given as 15.5 hours and its endurance at 1000 nmi radius from base is 10.5 hours. Using the range and speed data, endurance is calculated as:

E-6A Maximum Endurance: $E = {1.16(6350)}/{455} = 16.2$ (hours)

E-6A Endurance at 1000 nmi: $E = {1.16(6350-2000)}/{455} = 11.1$ (hours)

These are close but optimistic by 4.5 and 5.7%, respectively. Calculations for several other transport-class jets indicates similar optimism, possibly due to practical aspects of engine operation under loiter conditions.

A caveat: these equations and rules-of-thumb assume loitering at the same altitude as the cruise was done. A better loiter may be obtained at a lower altitude. However, other real-world factors ignored in this method seem to ameliorate obliterate this effect.

Derivation of Method – Prop

For a propeller-powered aircraft, the thrust obtained from the propeller reduces with velocity while the actual fuel consumption of the engine remains essentially constant. Therefore, the thrust specific fuel consumption is degraded substantially with increased velocity. One can develop a version of each Breguet equation using the engine's power-specific fuel consumption (C_P) by substitution into equations (1) and (2), as provided in equations (10) and (11) below (for derivation see Raymer[1]).

Prop Range:

$$R = \frac{\eta_p}{C_p} \frac{L}{D} ln\left(\frac{Wi}{Wf}\right) \qquad (10)$$

Prop Endurance:

$$E = \frac{\eta_p}{VC_p} \frac{L}{D} ln\left(\frac{Wi}{Wf}\right) \qquad (11)$$

(where η_p is the propeller efficiency parameter equal to thrust power obtained divided by engine power employed)

To determine available loiter time given range, we can start with the loiter equation (11) and employ ratios to permit usage of the *L/D* and velocity from cruise conditions, again collecting terms. The large bracket at the right of equation (13) is the range (see equation 10), with aerodynamic and propulsion terms under range conditions.

$$E_{loiter} = \frac{\eta_p}{C_p}\left(\frac{1}{V_{cruise}}\right)\left(\frac{V_{cruise}}{V_{loiter}}\right)\left(\frac{L}{D}\right)_{cruise}\left(\frac{\left(\frac{L}{D}\right)_{loiter}}{\left(\frac{L}{D}\right)_{cruise}}\right) ln\left(\frac{Wi}{Wf}\right) \quad (12)$$

$$E_{loiter} = \left(\frac{V_{cruise}}{V_{loiter}}\right)\left(\frac{\left(\frac{L}{D}\right)_{loiter}}{\left(\frac{L}{D}\right)_{cruise}}\right)\left(\frac{1}{V_{cruise}}\right)\left\{\frac{\eta_p}{C_p}\left(\frac{L}{D}\right)_{cruise} ln\left(\frac{Wi}{Wf}\right)\right\} \quad (13)$$

$$E_{loiter} = \left(\frac{V_{cruise}}{V_{loiter}}\right)\left(\frac{\left(\frac{L}{D}\right)_{loiter}}{\left(\frac{L}{D}\right)_{cruise}}\right)\left\{\frac{R_{cruise}}{V_{cruise}}\right\} \quad (14)$$

Note that the power-specific fuel consumption (C_P) is assumed to change little with speed. Also, we have assumed that propeller efficiency is essentially unchanged from cruise to loiter. This is a good assumption provided the aircraft has variable pitch propellers and that it is not cruising at such a speed that there are substantial losses due to tip-Mach effects. Should either of these be substantially non-constant, the method can be adjusted to include ratios from cruise to loiter conditions.

We now need to estimate the velocity and *L/D* ratios. It can be theoretically shown (see Raymer[1]) that propeller-powered aircraft cruise optimally at their best L/D speed, but loiter slower at 76% of cruise speed. At this lower speed the aircraft is flying at a condition where the ratio $C_L^{3/2}/C_D$ is maximized. This loiter speed yields, in an accidental show of analytical symmetry, a loiter L/D that is 0.866 times the cruise (ie. optimal) L/D. Substituting these ratios into equation (14) yields equation (15).

$$E_{loiter} = 1.14\left\{\frac{R_{cruise}}{V_{cruise}}\right\} \quad \text{(approximation for most props)} \quad (15)$$

This 14% increase is close but not identical to the differently-derived 16% increase for a jet if the change in specific fuel consumption (*C*) is neglected.

A check of this method for propeller-powered aircraft used the General Atomics Predator, which has 24 hours of loiter at 434 nmi radius and a quoted maximum endurance of 40 hours. Using equation (15) with the Predator best cruise speed of 70 kts gives:

Predator Endurance: $$E = {1.14(868)}/{70} = 14 \qquad \text{(hours)}$$

Adding this to 24 hours gives 38 hours, 5% less than the quoted value.

As another test, a notional piston-prop light twin previously designed by this author for optimization research[3] was evaluated. This design has an as-drawn takeoff gross weight of 2200 lbs {998 kg} and a span of 29 ft {8.8 m}. As-drawn range is calculated[*] to be 2585 nmi at a best speed of 140 kts. Using this simplified method gives a calculated loiter time of 21 hours, versus a refined loiter calculation of 20 hours (giving a 5% error).

Notional Light Twin Endurance: $$E = {1.14(2585)}/{140} = 21 \qquad \text{(hours)}$$

A turboprop has substantially more variation in fuel consumption with velocity than a piston-prop. To assess this, the Grumman E-2C Prowler was considered. Its range is given[2] as 1394 nmi at 268 kts, and endurance is 6 hours. By this method we get;

E-2C Endurance: $$E = {1.14(1394)}/{268} = 5.9 \qquad \text{(hours)}$$

This indicates that perhaps even a turboprop aircraft can be approximated by the propeller method above. Since the jet equation is slightly over-predicting endurance in our test cases, it is suggested that the 1.14 constant be used for jets as well.

Figure 1 shows the range and endurance of a number of aircraft plotted versus the approximation of equation 15 for various cruise speeds. Aircraft shown include the F/A-18, P-3, S-3, Global Hawk, Harrier, E-2C, and C-2, selected because both range and loiter information was available in public sources[2]. Also included is the notional light twin described above. This figure shows a fairly good correspondence, with the given data points lying fairly near their equivalence line for their cruise speed.

[*] Using the RDS-Professional aircraft design and analysis computer program

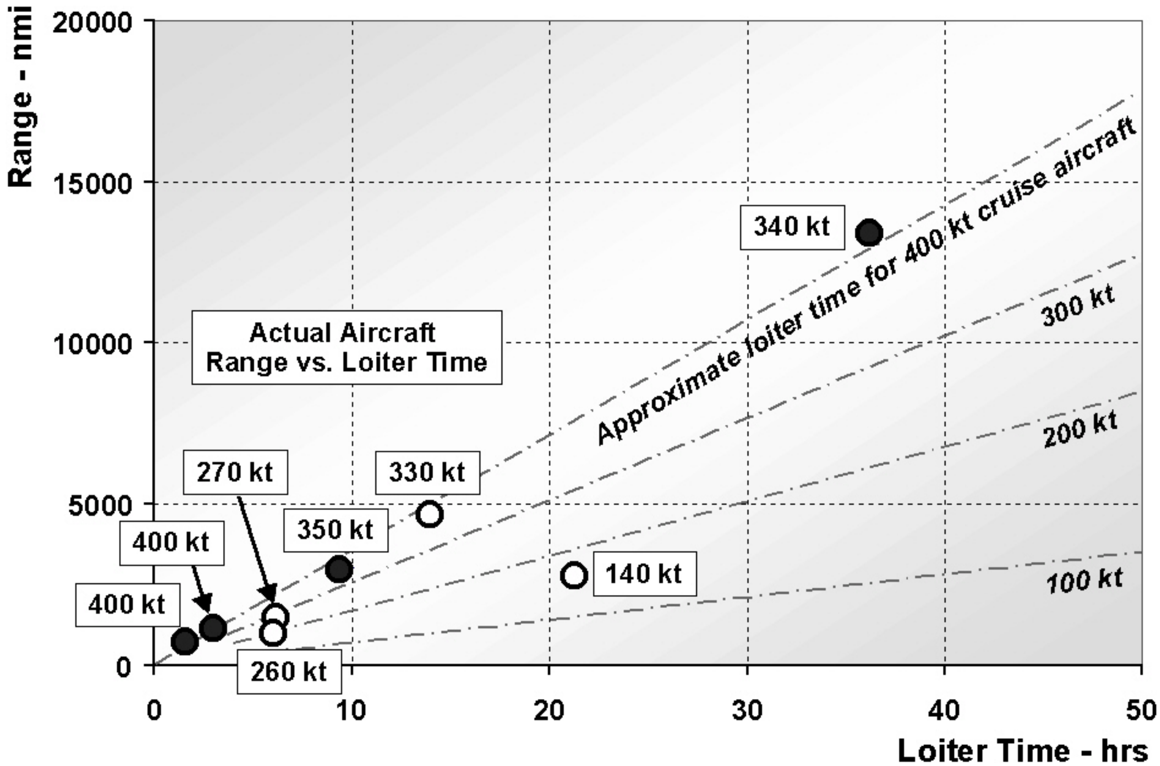

Fig. 1 - Actual Aircraft Range & Loiter Plotted against Approximation Results
(filled circles are jets, open circles are props)

The only exception is the Global Hawk (see data point with greatest range and loiter). Global Hawk flies at extreme altitude where its maximum speed and its stall speed are almost the same, so it cannot slow down for more-optimal loiter. Its loiter time is just slightly greater than range divided by cruise speed, without the 14% adjustment suggested herein.

Summary

A relationship between range and endurance was derived, based on the Breguet range and loiter equations. Given a known aircraft range and the cruise speed, equivalent loiter time can be estimated by a simple and useful rule of thumb:

"Loiter time equals range divided by cruise velocity, increased 14%"

Despite different derivations, this approximation can be used for both jets and props. Checks of this method were made which seem to indicate reasonable prediction of loiter time based on publicly available data, with results typically within 5% of the correct values.

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[1] Raymer, D., *AIRCRAFT DESIGN: A Conceptual Approach*, American Institute of Aeronautics and Astronautics, Washington, D.C., Third Edition 1999

[2] Taylor, J., *Jane's All the World Aircraft*, Jane's, London, UK, (1976, 1992, 2000)

[3] Raymer, D., *Enhancing Aircraft Conceptual Design using Multidisciplinary Optimization*, Ph.D. Thesis, Swedish Royal Institute of Technology, Stockholm, Sweden, 2002